Abstract

We study the Borsuk-Ulam theorem for the triple $(M, \tau, \mathbb{R}^n)$, where M is a compact, connected, 3-manifold equipped with a fixed-point-free involution τ. The largest value of n for which the Borsuk-Ulam theorem holds is called the $\mathbb{Z}_2$-index and in our case it takes the value 1, 2 or 3. We fully discuss this index according to cohomological operations applied on the characteristic class $x \in H^1(N, \mathbb{Z}_2)$, where $N = M/\tau$ is the orbit space.

In the oriented case, we obtain an expression of the index from the linking matrix of a surgery presentation of the orbit space. Recall that any oriented compact 3-manifold can be given such a surgery presentation. We apply our results to a few families of examples. As a warm up we first consider the double cover of lens spaces. We then fully discuss double covers of mapping tori. We consider the case of surgery presentations on algebraically split links. We finally study all free involutions on $S^1 \times S^2$, which include a non oriented one. We prove the Borsuk-Ulam theorem for the non oriented 3-Klein bottle K^3 with a natural involution.

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